DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE.

End Semester Examination – Winter 2018.

Course : B. Tech — **Sem. I**

Subject Name : Engineering Physics — **Subject Code : PHY1202**

Max. Marks : 60 — **Date 13/12/2018** — **Duration: 3 Hrs.**

Instructions:

1. All the questions are compulsory.

2. The level question/expected answer as per OBE or the Course Outcome (CO) on which the question is based is mentioned in ( ) infront of the question.

3. Use of non-programmable scientific calculators is allowed.

4. Assume suitable data wherever necessary and mention it clearly.

		Level/CO	Marks
Q.1	**Solve any two of the following**		
A)	In case of forced vibration, prove that $$A = \frac{f}{\sqrt{(\omega^2 - P^2)^2 + 4b^2p^2}}$$	CO1	6
B)	Explain Pizoelectric effect and Magnetostriction effect. What will be the Young's modulus of quartz plate if 5.5 mm thick quartz is used to produce an ultrasonic waves of frquency 0.4999 MHz. The density of the quartz is 2.65×10^3 kg/m^3.	CO1	6
C)	Explain with diagrams different types of polarization in dielectrics.	CO6	6
Q.2	**Solve any two of the following**		
A)	In case of wedge shaped film, prove that $\beta = \lambda / 2\theta$.	CO2	6
B)	Explain the principle and working of He-Ne Laser.	CO2	6
C)	i. A 20 cm long glass tube filled with a sugar solution of 15 gm of cane sugar in 100 cc of water is kept in the path of polarized light . Calculate the angle of rotation of cane sugar, specific rotation of cane sugar is 66^0 .	CO3	3
	ii. Calculate the refractive index of core and cladding of an optical fiber such that the numerical aperture of fiber is 0.27 and relative refractive index is 0.015.	CO3	3
Q.3	**Solve any two of the following**		
A)	With neat diagram explain the construction and working of G.M. Counter.	CO3	6
B)	What is Heseinberg's Uncertainty Principle? If the uncertainty in position of an electron is 4×10^{-10} m. Calculate the uncertainty in its momentum	CO3	6
C)	Derive Schrodinger's time independant wave equation	CO3	6
Q.4	**Solve any two of the following.**		
A)	Deduce the relation between interplaner spacing d and lattice constant a. Calculate the interplaner spacing for a (311) plane in a simple cubic lattice whose lattice constant is 2.109×10^{-10} m.	CO4	6
B)	State and prove Moseley's law. What is its importance?	CO4	6

C)	Derive an expression for electrmagnetic wave in free space and find the value of velocity of light in free space.	CO6	6

Q.5 Solve the following.

A)	Whar are magnetic domain and domain wall? Explain the B-H curve based on domain theory.	CO5	6
B)	Derive an expression for conductivity of a conductor interms of relaxation time of electron.	CO3	6

End

Dr. BATU, Lonere Raigad.
End semester Exam - Winter 2018
B. Tech F. Y. (All Branches)
Engineering Physics (PHY 1202)
Model solution

Q. 1)

A) In case of forced vibration, P.T. $A = \frac{f}{\sqrt{(\omega^2 - p^2)^2 + 4b^2p^2}}$

$\Rightarrow$ Forced vibration can be defined as the vibratn in which the body vibrates with a frequency other than its natural frequency under the action of an external periodic force.

The forces acted upon the particle are-

i) A restoring force proportional to the displacement but oppositely directed, given by $-\mu y$.

ii) A frictional force proportional to velocity but oppositely directed, given by $-r\frac{dy}{dt}$, where r is the frictional force per unit velocity &

iii) The external periodic force, represented by $F\sin pt$ where F is the maximum value of this force & $P/2\pi$ is its freqn.

So the total force acting on the particle is given by,

$-\mu y - r\frac{dy}{dt} + f\sin pt$

By Newton's 2nd law of motion this must be equal to the product of mass m of the particle & its acc^n

i.e. $m\,d^2y/dt^2$, hence,

$$m\frac{d^2y}{dt^2} = -\mu y - r\frac{dy}{dt} + f\sin pt$$

$$\therefore m\frac{d^2y}{dt^2} + r\frac{dy}{dt} + \mu y = f\sin pt$$

$$\therefore \frac{d^2y}{dt^2} + \frac{r}{m}\frac{dy}{dt} + \frac{\mu}{m}y = \frac{f}{m}\sin pt$$

$$\therefore \frac{d^2y}{dt^2} + 2b\frac{dy}{dt} + \omega^2 y = f\sin pt \quad - (1)$$

where $\frac{r}{m} = 2b$, $\frac{\mu}{m} = \omega^2$ & $\frac{f}{m} = f$

eq^n (1) is the differential eq^n of the motion of the particle

In this case, when the steady state is setup, the particle vibrates with the freq^n of applied force & not with its own natural freq^n. The sol^n of diff. eq^n (1) must be of the type,

$$y = A\sin(pt - \theta) \quad (2)$$

where A is the steady amplitude of vibrations & θ is the angle by which the displacement y lags behind the applied force $f\sin pt$,

A & θ being arbitrary const.

Diff. eq^n (2) we have,

$$\frac{dy}{dt} = Ap\cos(pt-\theta)$$

$$\therefore \frac{d^2y}{dt^2} = -Ap^2\sin(pt-\theta)$$

substituting these values in eq^n (1) we get,

$$-Ap^2\sin(pt-\theta) + 2bAp\cos(pt-\theta) + \omega^2 A\sin(pt-\theta) = f\sin pt$$

$$\therefore -Ap^2\sin(pt-\theta) + 2bAp\cos(pt-\theta) + \omega^2 A\sin(pt-\theta) = f\sin[(pt-\theta)+\theta]$$

$$\therefore A(\omega^2-p^2)\sin(pt-\theta) + 2bAp\cos(pt-\theta) = f\sin(pt-\theta)\cos\theta + f\cos(pt-\theta)\sin\theta$$

If this relation holds good for all values of t the coe. of $\sin(pt-\theta)$ & $\cos(pt-\theta)$ terms on both sides of this eq^n must be equal i.e. comparing the coe. of $\sin(pt-\theta)$ & $\cos(pt-\theta)$ on both sides, we have,

$$A(\omega^2-p^2) = f\cos\theta \quad (3)$$

$$\& \quad 2bAp = f\sin\theta \quad (4)$$

squaring eq^n (3) & (4)

$$A^2(\omega^2-p^2)^2 + 4b^2A^2p^2 = f^2$$

$$\therefore A^2[(\omega^2-p^2)^2 + 4b^2p^2] = f^2$$

$$\therefore \boxed{A = \frac{f}{\sqrt{(\omega^2-p^2)^2+4b^2p^2}}}$$

B) Piezoelectric effect :-

When certain crystals like quartz, rochelle salt, tourmaline etc. are stretched or compressed along certain axis, an electric potential difference is produced along a ⊥lar axis. This is known as piezoelectric effect.

Magnetostriction effect :-

When a rod of ferromagnetic material such as iron or nickel, is kept in a magnetic field ||el to its length, the rod suffers a change in its length. The change in length is independent of the dirⁿ of the magnetic field + depends only on the mag. of the field + nature of the material. This phenomenon is known as magnetostriction effect.

Numerical -

Given,

$t = 5.5\ mm = 5.5\times10^{-3}\ m \qquad Y = ?$

$\eta = 0.4999\ MHz = 0.4999\times10^{6}\ Hz$

$\rho = 2.65\times10^{3}\ kg/m^3$

$$\eta = \frac{k}{2t}\sqrt{\frac{Y}{\rho}}$$

$$Y = 4t^2\rho\eta^2$$

$$= 4\times(5.5\times10^{-3})^2\times2.65\times10^{3}\times(0.4999\times10^{6})^2$$

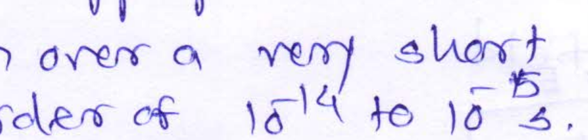
$$Y = 8\times10^{10}\ N/m^2$$

B

c) Types of polarization in dielectrics

1) Electric polarization:-

This is the polarization that results from the displacement of e^- clouds of atoms, molecules & ions with respect to heavy fixed nuclei to a distance that is less than the dimensions of the atoms, molecules or ions. It occurs in all dielectrics for any state of aggregation. The electric polarization sets in over a very short period of time, of the order of 10^{-14} to 10^{-15} s. It is independent of temp.



No field ← applied field.

(a) (b)

(a) atoms are not polarized in the absence of field.

(b) Electric polarizatⁿ results from the distortion of electric could by an applied field.

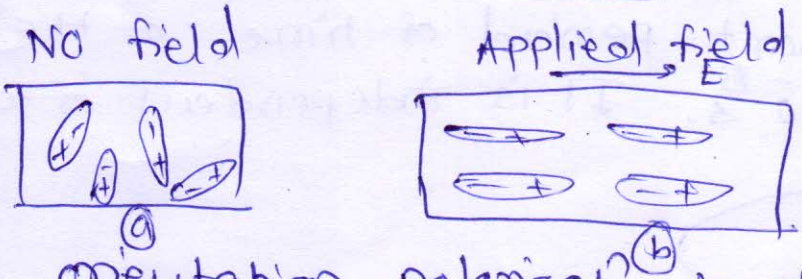
2) Ionic polarization:-

Ionic polarizatⁿ occurs in ionic crystals. It occurs due to the elastic displacement of +ve & -ve ions from their equilibrium positⁿ.

eg - sodium chloride crystal.

A NaCl crystal consists of Na^+ ions bound to Cl^- ions thro' ionic bond. If the interatomic distance is d, the molecule exhibits an intrinsic dipole moment equal to 'ed' when a dc electric field is applied to the molecule, the sodium + chloride ions are displaced in opposite dirⁿ until ionic bonding forces stop the process. The dipole moment of the molecule increases consequently, when the field dirⁿ is reversed the ions move closer & again the dipole moment undergoes a chang. Thus dipoles are induced. It occurs at a freqⁿ of 10^{13} Hz.

3) Orientation polarizatⁿ:-



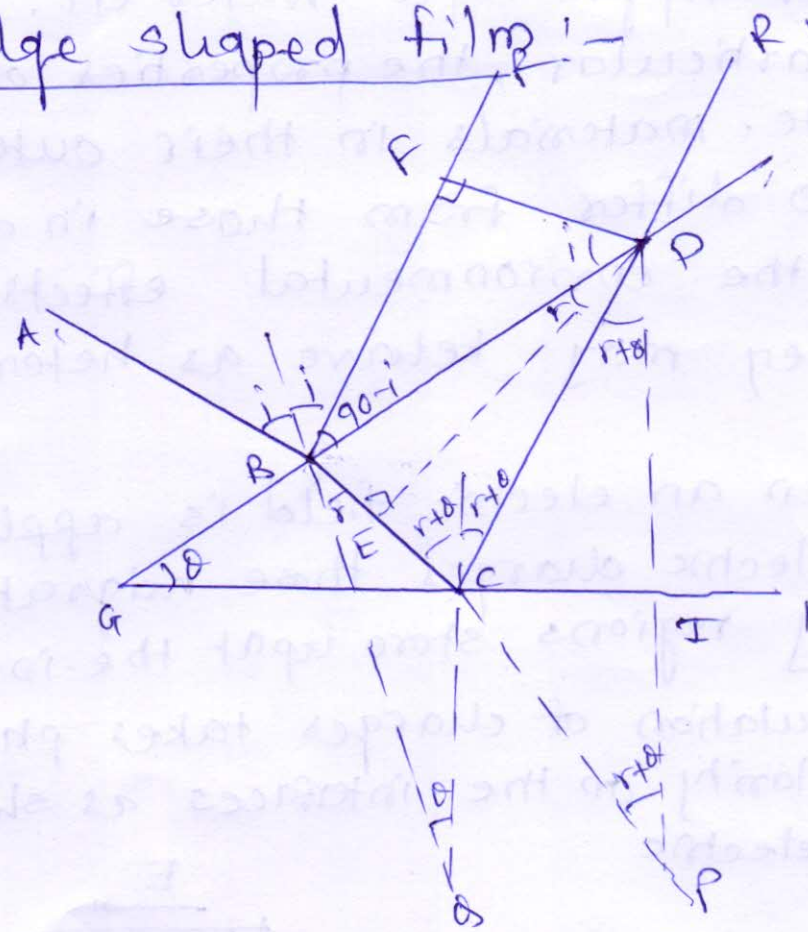
Orientation polarizatⁿ is characteristics of polar dielectrics, which consists of molecules having permanent dipole moment. In the absence of field the orientation of dipoles is random resulting in a complete cancellation of each other's effect as shown in fig (a)

When field is impressed the molecular dipoles rotate about their axis of symmetry to align with the applied field.

4) Space charge polarization :-

Space charge polarization occurs in heterogeneous dielectric materials in which there is a change of electrical properties between different phases + in the homogeneous dielectrics that contain impurities, pores filled with air, Q.2

A) Wedge shaped film :-



consider two plane surfaces, GH + GH, inclined at an angle θ + enclosing a wedge shaped air film. The thickness of air film increases from G to H as shown in fig.

Let μ be the refractive index of the material of the film. When this film is illuminated by sodium light, then the interference bet^n two systems of rays, one reflected from the front surface + the other obtained by internal reflection at the back surface + consequent transmitting at first surface, takes place. Path diff. bet^n two rays

$\Delta = \mu(BC + CD) - Bf$

but $\mu = \frac{\sin i}{\sin r}$, but $\sin i = \frac{Bf}{BD}$ + $\sin r = \frac{BE}{BD}$

$\therefore \mu = \frac{Bf}{BE}$ $\therefore Bf = \mu BE$

$\therefore \Delta = \mu(BC + CD) - \mu(BE)$

$\therefore \Delta = \mu(BE + EC + CD) - \mu BE \quad [\because BC = BE + EC]$

$\therefore \Delta = \mu(EC + CP) \quad [CD = CP]$

$\therefore \Delta = \mu EP \quad -①$

using ΔEDP

$\cos(r+\theta) = \dfrac{PE}{PD}$

$\therefore PE = PD\cos(r+\theta)$

$\therefore PE = 2t\cos(r+\theta) \quad -② \quad [PD = 2t]$

using eqⁿ ① + ②

$\Delta = 2\mu t\cos(r+\theta)$

Due to reflection additional phase change of $\lambda/2$ is introduced, hence,

$\Delta = 2\mu t\cos(r+\theta) \pm \lambda/2$

For destructive Interference,

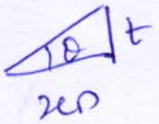
$\Delta = (2n+1)\lambda/2$

$\therefore 2\mu t\cos(r+\theta) + \dfrac{\lambda}{2} = (2n+1)\dfrac{\lambda}{2}$

$\therefore 2\mu t\cos(r+\theta) = n\lambda \quad -③$

For normal incidence, $r = 0$, $\mu = 1$

$\therefore 2t\cos\theta = n\lambda \quad -④$



t is the thickness of corresponding two n^{th} band.

$\therefore \tan\theta = \dfrac{t}{x_n}$

$\therefore t = x_n\tan\theta \quad -⑤$

put value of t in eqⁿ ④ we get,

$\therefore 2x_n \tan\theta \cdot \cos\theta = n\lambda$

$\therefore 2x_n \frac{\sin\theta}{\cos\theta} \cdot \cos\theta = n\lambda$

$\therefore 2x_n \sin\theta = n\lambda \quad -(6)$

For $(n+1)^{th}$ no. distance is x_{n+1}

$\therefore 2x_{n+1} \sin\theta = (n+1)\lambda \quad -(7)$

eq^n (7) $-$ (6) we get,

$2(x_{n+1} - x_n)\sin\theta = \lambda$

but, $x_{n+1} - x_n = \beta$

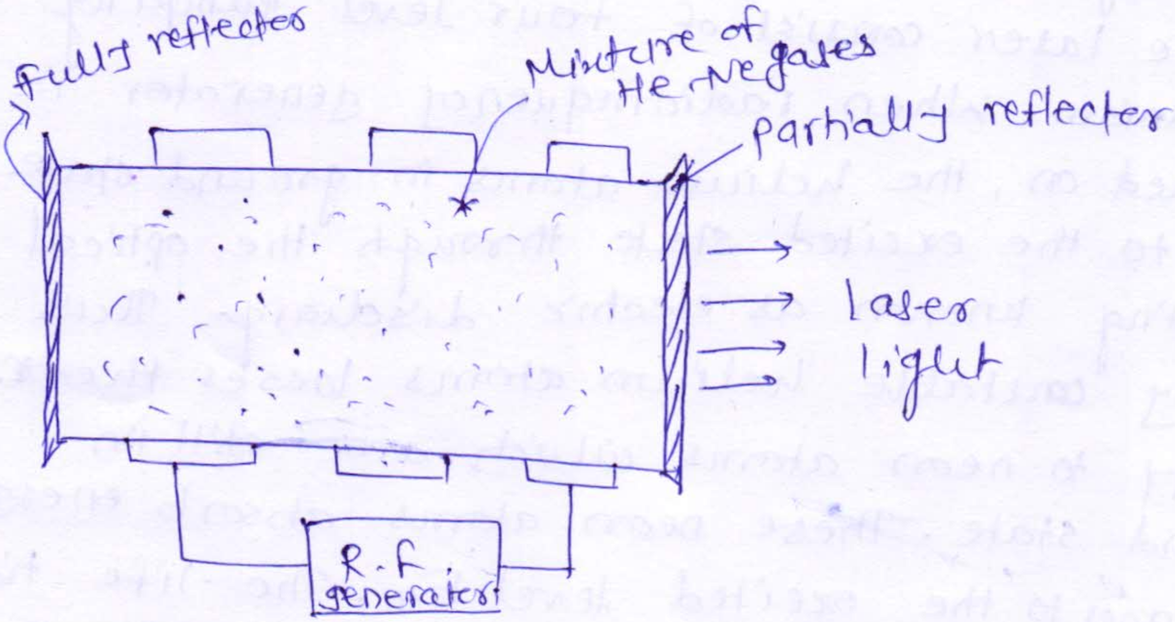
$\therefore 2\beta\sin\theta = \lambda$

$\therefore \beta = \frac{\lambda}{2\sin\theta}$

but, $2\sin\theta \simeq 2\theta$

$$\therefore \boxed{\beta = \frac{\lambda}{2\theta}}$$

B) He-Ne laser:-



Construction:-

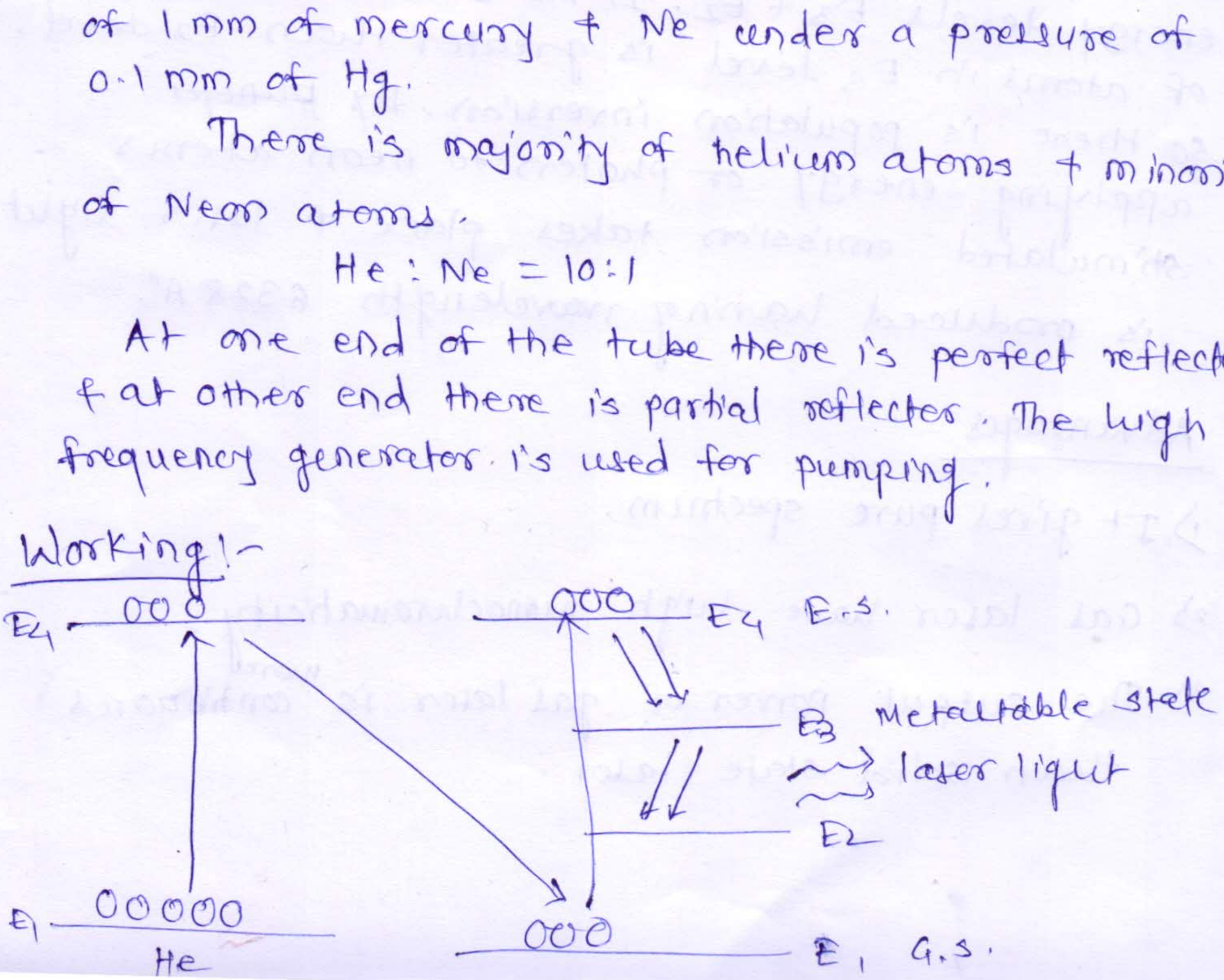
The gas laser consist of fused quartz tube with length 80 cm + diameter 1.5 cm. This tube is filled with mixture of He gas under a pressure of 1 mm of mercury + Ne under a pressure of 0.1 mm of Hg.

There is majority of helium atoms + minority of Neon atoms.

$$He : Ne = 10 : 1$$

At one end of the tube there is perfect reflector + at other end there is partial reflecter. The high frequency generator is used for pumping.

Working:-



Working:-

He-Ne laser consist of four level pumping mechanism. When radiofrequency generator is switched on, the helium atoms in ground state reach to the excited state through the optical pumping known as electric discharg. This highly unstable helium atoms looses their energy to neon atoms which are still in ground state. These neon atoms absorb energy and goes to the excited level E_4. The life time of excited state is very less so neon atoms are radially transmitted to the metastable state E_3 through rapid decay.

Now the probability is check betn two energy levels E_3 & E_2. It is observe that no. of atoms in E_3 level is greater than E_2 level. so there is population inversion. By ~~pumpi~~ applying energy of photon to neon atoms stimulated emission takes place & laser light is produced having wavelength 6328 A°.

Advantages:-

1) It gives pure spectrum.

2) Gas laser have high monochromaticity.

3) The output power of gas laser is more continuous than solid state laser.

c) i) Numerical

Given – $l = 20$ cm $\theta = ?$

$c = \frac{15}{100}$ $s = 66^\circ$

$s = \frac{10\theta}{lc}$

$\theta = \frac{s\,lc}{10} = \frac{66 \times 22 \times 0.15}{10}$

$\boxed{\theta = 19^\circ 8'}$

ii) Numerical

$n_1 = ?$

$n_2 = ?$

$NA = 0.27$

$\Delta = 0.015$

$NA = n_1\sqrt{2\Delta}$

$\therefore n_1 = \frac{NA}{\sqrt{2\Delta}} = \frac{0.27}{\sqrt{2 \times 0.015}}$

$\therefore \boxed{n_1 = 1.5588}$

$NA = \sqrt{n_1^2 - n_2^2}$

$\therefore NA^2 = n_1^2 - n_2^2$

$\therefore n_2^2 = -NA^2 + n_1^2$

$\therefore n_2^2 = -(0.27)^2 + (1.5588)^2$

$n_2^2 = 2.3569$

$\boxed{n_2 = 1.5352}$

3) A) G.M. Counter :-

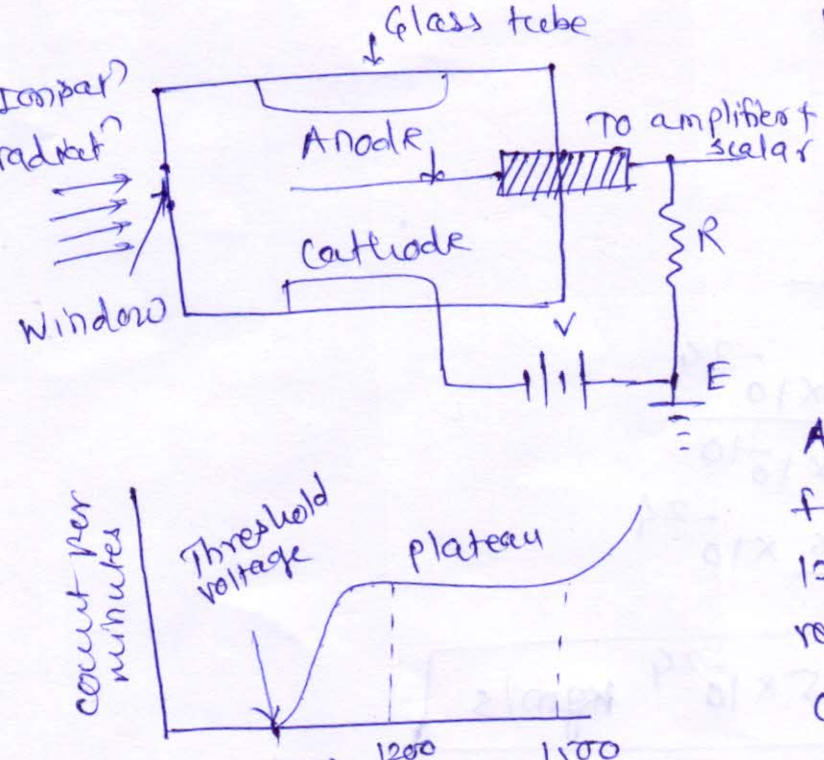
It consist of a fine wire placed along the axis of a hollow metal cylinder electrode enclosed in a thin glass tube. The tube contains a mixture of 90% argon at 10 cm pressure + 10% ethyl alcohol vapour at 1 cm pressure. Different mixtures of gases at diff. pressures are used in diff. designs. At one end of the tube, a window covered with thin mica sheet. is provided thro' which the ionizing particles or radiations may enter the tube. A d.c. potential of both 1200 volts is applied betn the cathode + the wire which acts as an anode. The value of the voltage is adjusted to be somewhat below the breakdown voltage of the gaseous mixture.

When a charged particle passes thro' the counter, it ionizes the gas molecules. The central wire attracts the electrodes while the cylindrical electrode attracts the positive ion. This cause an ionization current which depends upon the applied voltage. At sufficiently high voltages, the electrons gain high kinetic energy + causes further ionization of argon atoms. Thus the larger no. of secondary electrons is independent of the no. of primary ions produced by incomping particle due to the following reasons:-

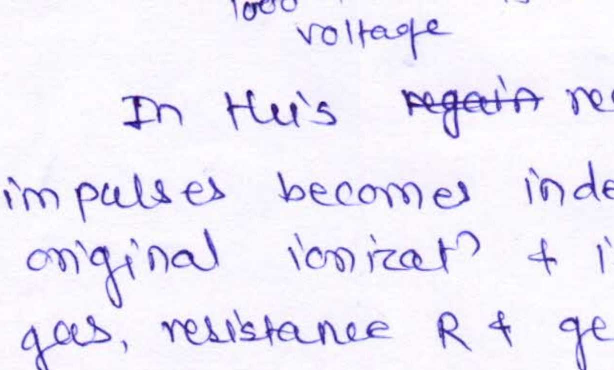
i) The production of secondary electrons is not confined to the region near the primary electrons but it takes place all along the length of the wire as their no. of extremely large ($\approx 10^8$).

ii) The production of secondary electrons at one pt. affects the production at other pts.

The sucessful operation of G.M. counter depends upon the proper voltage to the electrodes. Fig. represents the counts per minute as a fn of voltage. It is obvious from the fig. that if the voltage is less than 1000 volts, there is no discharge i.e. no secondary ionizatn.



When the voltage is increased secondary ionizatn takes place. Now the no. of impulses increases almost linearly with applied voltage. As the applied voltage is further increased to about 1200 volts the no. of impulses remains const. over a certain region known as plateau.



In this ~~regain~~ region, the magnetitude of impulses becomes independent of the amount of original ionizatn & is a fn of potential, nature of gas, resistance R & geometrical condition of apparatus. This region is most suitable of G.M. counter.

B) Heisenberg's uncertainty principle:-

It is impossible to specify precisely & simultaneously the value of both members of particular pairs of physical variables that describe the behaviour of an atomic system.

Let uncertainty in position $= \Delta x$ &
uncertainty in momentum $= \Delta p$

$\therefore \Delta x \cdot \Delta p \simeq h$

Numerical

Given:-

$\Delta x = 4 \times 10^{-10}$ m $h = 6.6 \times 10^{-34}$

$\Delta p = ?$

$\Delta x \cdot \Delta p = h$

$$\Delta p = \frac{h}{\Delta x}$$

$$= \frac{6.6 \times 10^{-34}}{4 \times 10^{-10}}$$

$$= \frac{6.6}{4} \times 10^{-24}$$

$$\therefore \boxed{\Delta p = 1.65 \times 10^{-24} \text{ kgm/s}}$$

Schrodinger's time independant wave eq^n :-

According to debroglie theory,

$$\lambda = \frac{h}{mv}$$

The classical diff. eq^n of a wave motion is given by,

$$\frac{\partial^2 \psi}{\partial t^2} = v^2 \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right)$$

but,

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \nabla^2$$

$$\therefore \frac{\partial^2 \psi}{\partial t^2} = v^2 \nabla^2 \psi \quad \text{---} (1)$$

where $\nabla^2 \rightarrow$ Laplacian operater

$v \rightarrow$ wave velocity.

The solution of eq^n (1) is

$$\psi(x, y, z, t) = \psi_0(x, y, z) \cdot e^{-i\omega t}$$

$$\psi(r, t) = \psi_0(r) \cdot e^{-i\omega t}$$

but $r = (x, y, z)$

$$\therefore \psi = \psi_0 e^{-i\omega t} \quad \text{---} (2)$$

diff. eq^n (2) twice, we get,

$$\frac{\partial \psi}{\partial t} = (-i\omega) \psi_0 e^{-i\omega t}$$

$$\therefore \frac{\partial^2 \psi}{\partial t^2} = (-i\omega)(-i\omega) \psi_0 e^{-i\omega t}$$

$$\therefore \frac{\partial^2 \psi}{\partial t^2} = i^2 \omega^2 \psi_0 e^{-i\omega t}$$

but $i^2 = -1$

$$\frac{\partial^2\psi}{\partial t^2} = -\omega^2 \psi_0 e^{-i\omega t}$$

but from ②

$$\frac{\partial^2\psi}{\partial t^2} = -\omega^2 \psi \quad - ③$$

By using eqⁿ ① + ③ we get,

$$v^2 \nabla^2 \psi = -\omega^2 \psi$$

$$\therefore \nabla^2\psi + \frac{\omega^2}{v^2}\psi = 0 \quad - ④$$

but $\omega = 2\pi\nu$

$\therefore \nu = \frac{c}{\lambda} = \frac{v}{\lambda}$ $\quad \therefore \omega = \frac{2\pi v}{\lambda}$ $\quad \therefore \frac{\omega}{v} = \frac{2\pi}{\lambda}$

Put this value in eqⁿ ④ we get,

$$\nabla^2\psi + \frac{4\pi^2}{\lambda^2}\psi = 0 \quad - ⑤$$

but $\lambda = \frac{h}{mv}$ $\quad \therefore \frac{1}{\lambda} = \frac{mv}{h}$

Put this value in eqⁿ ⑤

$$\therefore \nabla^2\psi + \frac{4\pi^2 m^2 v^2}{h^2}\psi = 0 \quad - ⑥$$

If E & V be the total & potential energies of the particle respectively. Then K.E. $\frac{1}{2}mv^2$ is given by,

$$\frac{1}{2}mv^2 = E - V$$

$\therefore \frac{m^2v^2}{2m} = E - V$ $\quad \therefore m^2v^2 = 2m(E-V)$

put this value in eqⁿ ⑥

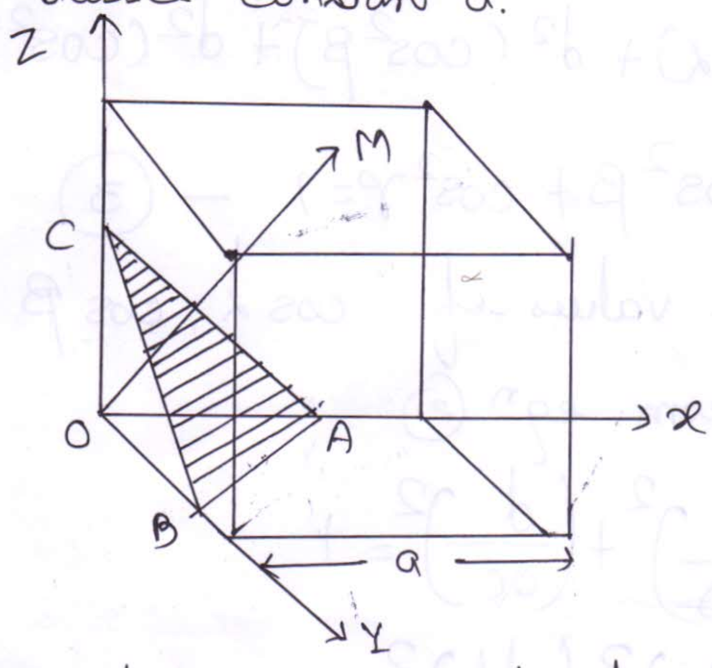
$$\therefore \nabla^2\psi + \frac{4\pi^2\, 2m\,(E-V)}{h^2}\psi = 0$$

$$\therefore \boxed{\nabla^2\psi + \frac{8\pi^2 m\,(E-V)}{h^2}\psi = 0}$$ also, $$\boxed{\nabla^2\psi + \frac{2m(E-V)}{\hbar^2}\psi = 0}$$

This is schrodinger's time independent wave equation.

Q 4 A Solve any two of the following.

A) Deduce the relation between Interplanar spacing d and lattice constant a.



Consider the case of plane ABC of a cubic crystal as shown in fig. This plane belongs to a family of planes whose Miller indices are hkl because Miller indices represent a set of planes. Here ON is the perpendicular drawn from the origin to this plane. The distance ON represents the interplanar spacing d of the family of the planes. Let α, β and γ be the angles between coordinate axes x, y, z and ON respectively. The intercepts of the plane on the three axes are

$$OA = a, \quad OB = a \quad \text{and} \quad OC = \frac{a}{l}$$

where a is the length of the cube edge.

$$\cos\alpha = \frac{d}{OA}, \quad \cos\gamma = \frac{d}{OB} \quad \text{and} \quad \cos\beta = \frac{d}{OC}$$

$$ON = \left[(x)^2 + (y)^2 + (z^2)\right]^{1/2}$$

$$d = \left[d^2(\cos^2\alpha) + d^2(\cos^2\beta) + d^2(\cos^2\gamma)\right]^{1/2}$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \quad — (3)$$

Substituting the values of $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ in eqn (3) from eqn (2)

$$\left(\frac{d}{OA}\right)^2 + \left(\frac{d}{OB}\right)^2 + \left(\frac{d}{OC}\right)^2 = 1$$

$$\left(\frac{dh}{a}\right)^2 + \left(\frac{dk}{a}\right)^2 + \left(\frac{dl}{a}\right)^2 = 1$$

or $$\frac{d^2}{a^2}(h^2+k^2+l^2) = 1$$

$$d^2 = \frac{a^2}{(h^2+k^2+l^2)}$$

or $$\boxed{d = \frac{a}{\sqrt{(h^2+k^2+l^2)^2}}} \quad — (4)$$

Calculate the interplanar spacing for a (311) plane in a simple cubic lattice whose lattice constant is 2.109×10^{-10} m.

$$d = \frac{a}{\sqrt{(h^2+k^2+l^2)^2}}$$

$$= \frac{2.109 \times 10^{-10}}{\sqrt{((3^2+1^2+1^2))^2}}$$

$$= \frac{2.109 \times 10^{-10}}{\sqrt{(9+1+1)^2}} = \frac{2.109 \times 10^{-10}}{\sqrt{(11)^2}} = \frac{2.109 \times 10^{-10}}{11}$$

$$\boxed{d = 0.1912 \times 10^{-10} \text{ m}}$$

B) State and prove Mosley's law. What is its importance. [08]

Ans → The frequency of a spectral line of characteristic X-ray spectrum varies directly as the square of atomic number Z of the element emitting it. Mosley found the following facts:

i) The characteristic X-ray spectra of different elements are similar to each other in the sense that each consists of K, L and M series.

2) The frequency of lines produced by an element of higher atomic number is greater than that produced by an element of lower atomic number. The reason being that binding energy of electron increases as we go from one element to another. The relationship can be given by

$$\nu \propto (Z-a)^2$$

or $$\nu = b(Z-a)^2 \quad — (1)$$

where ν = frequency of characteristic radiation

b = constant which is different for different series.

a = constant known as screening constant.

Derivation of Mosley's Law → Consider the atom of an element with atomic number Z. According to Bohr's theory, the energy of an electron in an orbit of principal quantum number n_1 is given by

$$E_{n_1} = -\frac{mZ^2e^4}{8\epsilon_0^2 n_1^2 h^2} \quad — (2)$$

Similarly

$$E_{n_2} = -\frac{mZ^2e^4}{8\epsilon_0^2 n_2^2 h^2} \quad — (3)$$

Considering the effect of electrons which screen the positive charge of the nucleus reducing the value of Z to (Z-a) the above eqⁿ can be written as

$$E_{n_1} = -\frac{m(Z-a_1)^2 e^4}{8\epsilon_0^2 n_1^2 h^2} \quad (4)$$

$$E_{n_2} = -\frac{m(Z-a_2)^2 e^4}{8\epsilon_0^2 n_2^2 h^2} \quad (5)$$

$$E_{n_1} - E_{n_2} = \frac{me^4}{8\epsilon_0^2 h^2}\left(\frac{(Z-a_2)^2}{n_2^2} - \frac{(Z-a_1)^2}{n_1^2}\right),$$

when Z is very high $a_1 = a_2 = a$

$$\Delta E = E_{n_1} - E_{n_2} = \frac{me^4}{8\epsilon_0^2 h^2}(Z-a)^2\left[\frac{1}{n_2^2} - \frac{1}{n_1^2}\right]$$

$$\nu = \frac{\Delta E}{h} = \frac{me^4 (Z-a)^2}{8\epsilon_0^2 h^2}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right) \quad (6)$$

Considering the case of K_α we have $n_1 = 2$ and $n_2 = 1$

$$\nu_{K\alpha} = \frac{me^4}{8\epsilon_0^2 h^3}(Z-a)^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

or $\nu_{K\alpha} \propto (Z-a)^2$

Importance of Mosley's law → According to Mosley's law, it is the atomic number of an element and not the atomic weight which determines the physical and chemical properties of an element.

By considering the X-ray spectra of rare earth, this law has been helpful in determining their atomic numbers and also fixing their positions in periodic table.

This law has led to the discovery of some new elements like hafnium (72), promethium (61), technitium (43) etc.

---

c) Derive an expression for electromagnetic wave in free space and find the value of velocity of light in free space. $\boxed{06}$

Ans → By applying Maxwell's equation we can develop wave eqⁿs for transverse electric and magnetic fields in free space.

Let us consider a region where charge density ρ and current density j are both Zero. Maxwell's eqⁿs then reduces to

$$\nabla . E = 0 \quad — (1)$$

$$\nabla \times E + \frac{\partial B}{\partial t} = 0 \quad — (2)$$

$$\nabla . B = 0 \quad - (3)$$

and $$\nabla \times H = \frac{\partial D}{\partial t} \quad - (4)$$

Differentiating eqn (2) we get

$$\nabla \times \frac{\partial E}{\partial t} + \frac{\partial^2 B}{\partial t} = 0 \quad - (5)$$

from eqn (4)

$$\nabla \times \nabla \times H = \nabla \times \frac{\partial D}{\partial t} \quad (\text{as } D = \epsilon_0 E)$$

$$\nabla \times \nabla \times H = \epsilon_0 \nabla \times \frac{\partial E}{\partial t} \quad - (6)$$

Substituting from eqn (5)

$$\nabla \times \nabla \times H = -\epsilon_0 \frac{\partial^2 B}{\partial t^2}$$

or $$\nabla(\nabla . H) - \nabla^2 H = -\mu_0 \epsilon_0 \frac{\partial^2 H}{\partial t^2} \quad (\text{As } B = \mu_0 H)$$

Since $\nabla . B = 0$ ~~or~~ so $\nabla . H = 0$ from eqn (3)

$$\nabla^2 H = \mu_0 \epsilon_0 \frac{\partial^2 H}{\partial t^2} \quad - (7)$$

Similarly on differentiating both sides of eqn (4)

$$\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} \quad - (8)$$

It is obvious that eqns (7) and (8) show that H and E gives general wave eqn.

$$\nabla^2 u = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = 0 \quad - (9)$$

where $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

Now $\epsilon_0 = 8.854 \times 10^{-12}$ F/m.

$\mu_0 = 4\pi \times 10^{-7}$ H/m.

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.854 \times 10^{-12}}}$$

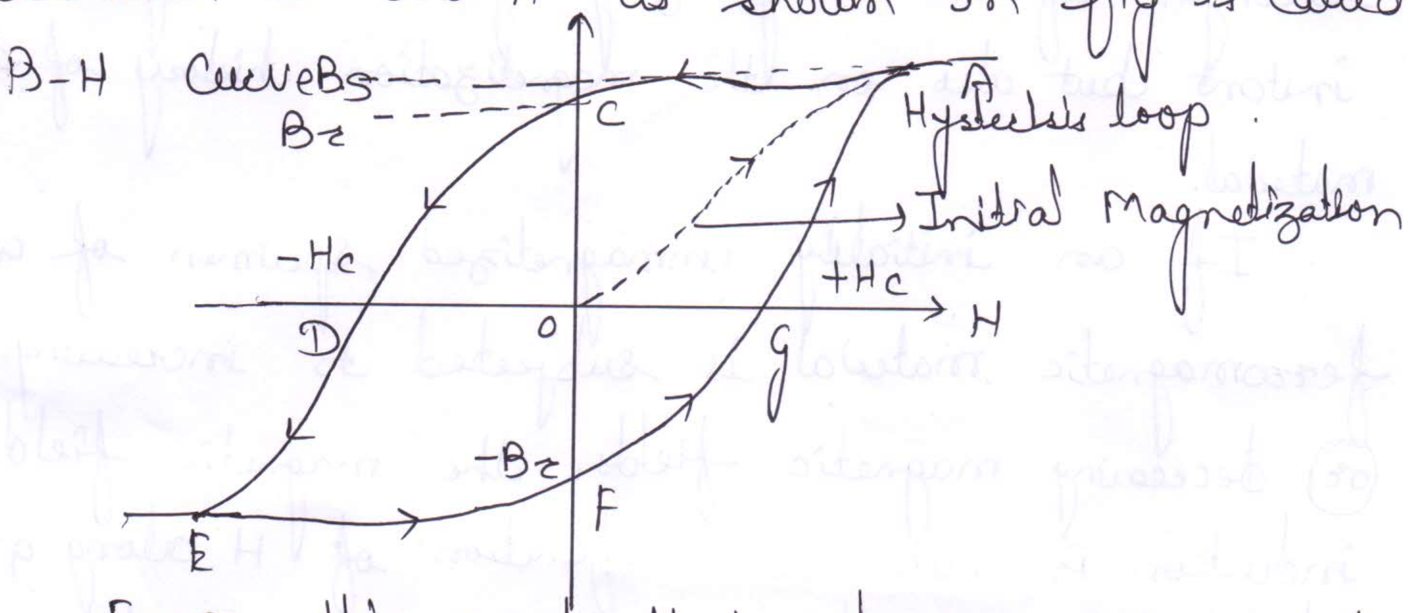
$$\boxed{c = 2.998 \times 10^{8} \text{ m/sec.}}$$

This is the speed of light in free space.

Q 5 Solve the following $\boxed{06}$

A) What are magnetic domain and domain walls? Explain the B-H curve based on domain theory.

→ The variation of flux density with magnetic field intensity is not linear. A graph plotted between B and H as shown in fig is called B-H Curve



From this graph it is clear that curve rises rapidly at first indicating big change in the value of B for a corresponding small change in H. The slope of the curve gradually decreases indicating a small change in B for increasing larger values of H. A stage is reached when the slope of the curve becomes constant and this condition is known as saturation. The units of B are taken Wb/m² and of H AT/m.

A Typical property of ferromagnetic materials is hysteresis. It is defined as the lag in the changes of magnetostriction behind variation of magnetic field. Because of hysteresis, the magnetization of ferromagnetic material depends not only on the strength of the magnetizing field at the given instant but also on the magnetization history of the material.

If an initially unmagnetized specimen of a ferromagnetic material is subjected to increasing or decreasing magnetic fields, the magnetic field induction B varies as a function of H along a closed loop, called the hysteresis loop. The curve begins at O.

As H is increased, the field B begins to increase slowly, then more rapidly and finally attaining a saturation value and becoming independent of H. The maximum value of B is the saturation flux density Bs and the corresponding magnetization is Ms.

If H is decreased, B also decreases but following a path AC instead of the original path AO. Thus B lags behind H. When H becomes zero

B does not become zero but has a value equal to OC. This magnetic flux density remaining in the material is called residual magnetism. It indicates that the material remain magnetized even in the absence of an external applied field H. The power of retaining the magnetism is called retentivity. Thus retentivity of a material is a measure of the magnetic flux density remaining in the material when the magnetizing field is removed.

If the magnetic field H is increased in the reverse direction, the value of B decreases along the path CD. It becomes zero, when H attains a value equal to OD. It means that to reduce magnetic induction within the material, a field of magnitude H_C must be externally applied in a direction opposite to that of the original magnet. ising field. H_C is called the coercivity. It is a measure of the magnetic field strength required to destroy the residual magnetism in the material.

As the applied field H is increased further, in the negative direction, saturation is ultimately reached in the reverse direction.

On reversing the variation of the field H, a curve similar to ACDE is traced through points EFGA, yielding a negative remanance and a positive coercivity H_c. At points C and F where the specimen is magnetized in the absence of any external magnetic field, it is said to become a permanent magnet.

The closed curve ACDEFGA represents a cycle of magnetization of the specimen and it is known as hysteresis loop of the specimen.

B) Derive an expression for conductivity of a conductor in terms of relaxation time of electron; $\boxed{06}$

→ The free electron theory was proposed by Drude in 1900 and later was extended by Lorentz. It was assumed that the free electrons move in a region of constant potential. This theory successfully explained Ohm's law and high electrical conductivity of metals.

When an electric field E is applied on a metal, the electrons in it experience a force $-Ee$. If v is the velocity of free electron and t is the average time between two consecutive collisions, the frictional force appearing the continuous acceleration of the electron may be written as $-\frac{mv}{t}$.

Using Newton's second law, the eqⁿ of motion may be written as

$$m\frac{dv}{dt} = -eE = -\frac{mv}{t}$$

where m is the mass of electron and E is the intensity of applied electric field.

Under steady state condition,

$\frac{dv}{dt} = 0$ and the electron attains a steady value of velocity v_d. Under steady state condition

$$0 = -eE - m\frac{v_d}{\tau}$$

$$-eE = m\frac{v_d}{\tau}$$

$$\boxed{v_d = -\frac{eE\tau}{m}} \quad — (1)$$

where v_d is the drift velocity. It causes current flow in a conductor.

Let n be the number of free electrons per unit volume of the conductor. n is called the free electron density or concentration.

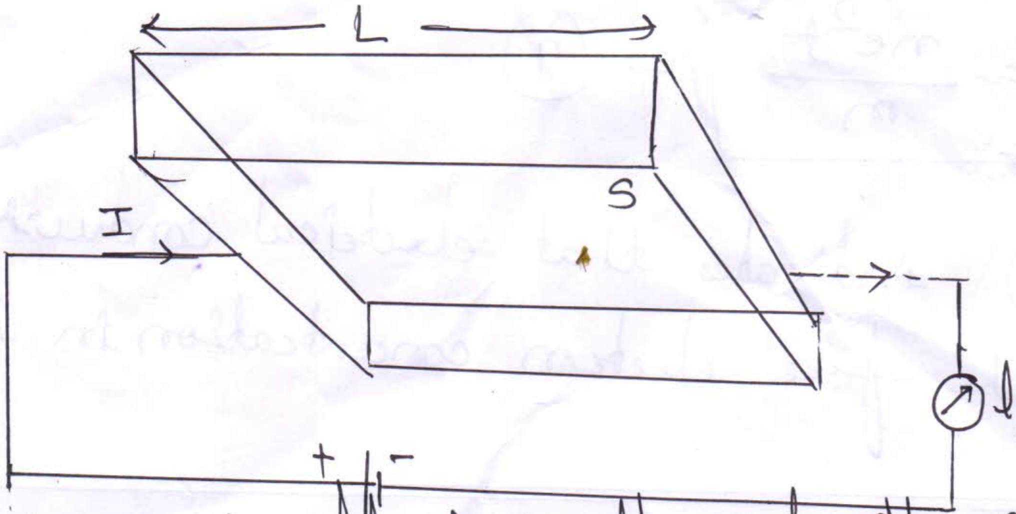
The total number of electrons in the metal is given by

$N =$ electrons per unit volume × Total volume

$$N = nAL \quad — (2)$$

The total charge present in the solid block

$$Q = Ne = nALe \quad — (3)$$



Current flowing through the solid is given by

$$I = \frac{Q}{t} = \frac{nALe}{t} \quad \text{--- (4)}$$

The term $\frac{L}{t}$ represents velocity and gives the average drift velocity v_d of electrons in the solid

$$I = neAv_d \quad \text{--- (5)}$$

The current density is defined as $J = \frac{I}{A}$

$$J = nev_d \quad \text{--- (6)}$$

$$J = ne\left(\frac{eE\tau}{m}\right) = \frac{ne^2\tau E}{m} \quad \text{--- (7)}$$

According to eq^n $\sigma = \frac{IL}{VA}$ S/m

As $\frac{V}{L} = E$ the above eq^n can be written as

$$\sigma = \frac{J}{E}$$

$$\boxed{J = \sigma E} \quad \text{--- (8)}$$

Equating R.H.S of eq^n (7) and (8)

$$\boxed{\sigma = \frac{ne^2\tau}{m}} \quad - (9)$$

Eqn (9) indicates that electrical conductivity depends on the free electron concentration in it.